Microeconomics 1A December 10, 2018

Final Exam (1 hour and 45 minutes)

Mobile phones, class notes and problem sets are strictly prohibited

Read and think before you write, and try to be both concise and precise

Exercise 1 (45 minutes). There are $L = 3$ commodities. The firm produces commodity 3 by using commodities 1 and 2 as inputs. The production function of the firm is given by

$$f(z_1, z_2) = \sqrt{(z_1 + z_2)} \quad \text{with } z_1 \geq 0 \text{ and } z_2 \geq 0$$

1) Write the cost minimization problem (CMP) of this firm.
2) Give the definitions of the demand of inputs and the cost function of this firm.
3) State the proposition on the non-emptiness of the demand of inputs. Then explain why the demand of inputs of this firm must be non-empty for every $(p_1, p_2) \in \mathbb{R}^2_{++}$ and for every output level.
4) State the proposition on the first order conditions (FOC) associated with the cost minimization problem (CMP).
5) Compute the demand of inputs and the cost function of this firm (**justify carefully your answers**).
6) Remind the relationship among the supply, the demand of inputs and the cost function of a firm. Then, determine the supply of this firm for every $(p_1, p_2, p_3) \in \mathbb{R}^3_{++}$.

Exercise 2 (20 minutes). Give the definitions of the following three properties for a general production set $Y \subseteq \mathbb{R}^L$: impossibility of free production (i.e., *no free lunch*), free-disposal, decreasing returns to scale.

Now there are $L = 3$ commodities. A firm produces two outputs, namely commodities 1 and 2, by using commodity 3 as an input. The transformation function of the firm is

$$t(y_1, y_2, y_3) = \alpha y_1 + \beta y_2 - 2\sqrt{-y_3} \quad \text{with } y_3 \leq 0$$

where the parameters α and β are strictly positive. Write the production set determined by this transformation function and show that this production set satisfies the three properties defined above.

Exercise 3 (40 minutes). Let $p = (p_1, ..., p_\ell, ..., p_L) \in \mathbb{R}^L_{++}$ be a price system and $w > 0$ be the wealth of the consumer. The demand of the consumer is $x(p, w) = (x_1(p, w), ..., x_\ell(p, w), ..., x_L(p, w)) \in \mathbb{R}^L_{++}$.

Assume that $x(p, w)$ is differentiable and satisfies Walras' Law.

1) A commodity $\ell \in \{1, ..., L\}$ is called *inferior* if its demand is strictly decreasing in wealth. Show that the L commodities cannot be all inferior commodities.
2) A commodity $\ell \in \{1, ..., L\}$ is called *luxury* if $D_w x_\ell(p, w) > \dfrac{x_\ell(p, w)}{w}$. Show that the L commodities cannot be all luxury commodities.
3) Show that if $x(p, w)$ is homogeneous of degree one with respect to w, then $D_w x_\ell(p, w) = \dfrac{x_\ell(p, w)}{w}$ for every commodity ℓ.
4) First, remind us why $x(p, w)$ is homogeneous of degree zero with respect to (p, w). Second, assume that:

i) $x(p, w)$ is homogeneous of degree one with respect to w, and

ii) for every commodity ℓ, the demand $x_\ell(p, w)$ of commodity ℓ does not depend on the price p_k for every commodity $k \neq \ell$.

Show that for every commodity ℓ, $x_\ell(p, w) = \dfrac{\alpha_\ell w}{p_\ell}$, where $\alpha_\ell > 0$ is a constant independent of (p, w).