

Final Exam (2 hours)

No documents or calculator authorised.

Exercise 1 (40 minutes). We consider a pure exchange economy with two commodities and two consumers:

- The preferences of consumer 1 are given by the utility function $u_1(x_{11}, x_{12}) = x_{11}^{0.5} x_{1,2}^{0.5}$
- The preferences of consumer 2 given by the utility function $u_2(x_{21}, x_{22}) = x_{21}^{0.25} x_{22}^{0.75}$
- The total resources of the economy are $r = (r_1, r_2) = (3, 3)$.

1. Draw the Edgeworth box and represent the indifference curves of both consumers.
2. Determine analytically the set of all Pareto optimal allocations.
3. Determine the Pareto optimal allocation (x^*, y^*) for which both consumers consume the same quantity of commodity 2.

Exercise 2 (40 minutes). We consider the exchange economy with 2 commodities and 2 consumers such that:

- Consumer 1 has utility $u_1(x_{1,1}, x_{1,2}) = \min(x_{1,1}, x_{1,2})$
- Consumer 2 has utility $u_2(x_{2,1}, x_{2,2}) = x_{2,1} + x_{2,2}$
- The total resources in the economy are $e = (4, 2)$

1. Prove that at a Pareto optimum consumer 1 consumes the same quantity of both goods.
2. Determine the set of Pareto Optima.

We then consider that a firm with production set $Y = \{(y_1, y_2) \in \mathbb{R}_+^2 \mid y_1 \leq 0, y_2 \leq -\alpha y_1\}$ is added to the economy.

- 3 Determine the set of feasible allocations.
- 4 Determine the set of Pareto optima assuming $\alpha < 1$.

Exercise 3 (40 minutes). We consider a production economy with 3 goods, 2 consumers and 2 firms (good 3 can be seen as labor/leisure and its price will be normalized to 1 throughout)

- Consumer 1 has initial endowment $e_1 = (0, 0, 1)$ and his preferences are given by the utility function $u_1(x_{11}, x_{12}, x_{13}) = x_{11}^{1/6} x_{1,2}^{5/6}$

- Consumer 2 has initial endowment $e_2 = (0, 0, 1)$ and his preferences are given by the Cobb-Douglas utility function $u_2(x_{21}, x_{22}, x_{23}) = x_{21}^{2/3} x_{22}^{1/3}$
 - Firm 1 has production set $Y_1 = \{(y_{1,1}, y_{1,2}, y_{1,3}) \in \mathbb{R}^3 \mid y_{1,3} \leq 0, y_{1,1} \leq -y_{1,3}\}$
 - Firm 2 has production set $Y_2 = \{(y_{2,1}, y_{2,2}, y_{2,3}) \in \mathbb{R}^3 \mid y_{2,3} \leq 0, y_{2,2} \leq \sqrt{-y_{2,3}}\}$
 - Consumer 1 receives all the profits of both firms
1. Determine the supply and profits of both firms.
 2. Determine the demand of both consumers.
 3. Determine the equilibrium of the economy.