Final exam "Optimization A". 11 december 2018

Teacher : Philippe Bich. Delay : 2H (10H-12H), no documents. **Every exit is definitive.**

All answers or affirmations have to be fully justified, i.e. if you use theorem, quote it, if you use precise properties, quote them and prove they are true, etc.

Exercise 1

We consider the maximization problem

$$(P) \max x + y - z$$

under the constraints $x^2 + y^2 = 1$ and $y + z = 1$, where x, y, z are reals (possibly negative).

1) Is the set of constraint C bounded ? closed ? prove every affirmation.

2) Prove that P has at least one solution.

3) Prove that the set of constraints satisfies the regularity condition that allows to use Lagrange theorem (we recall that this theorem is a version of KKT Theorem for which there are only equality constraints).

4) Write first order necessary conditions, and solve it.

5) Solve (P).

Exercise 2

We consider the maximization problem

$$(P) \max x^2 + 2y^2$$

under the constraints $x \geq 0$ and $y \geq 0$ $x \leq 3$ and $y \leq 3$ and $x + y \leq 2$, where x, y are reals.

1) Is the set of constraint C bounded ? closed ? make a picture of C.

2) Prove that P has at least one solution.

3) Prove that the solution of P cannot be interior to C.

4)

a) Find the maximum of $x^2 + 2y^2$ when $(x, y) \in C$ with the additional constraint $x = 0$. b) Find the maximum of $x^2 + 2y^2$ when $(x, y) \in C$ with the additional constraint $y = 0$. c) Find the maximum of $x^2 + 2y^2$ when $(x, y) \in C$ with the additional constraint $x = 3$. d) Find the maximum of $x^2 + 2y^2$ when $(x, y) \in C$ with the additional constraint $y = 3$. e) Find the maximum of $x^2 + 2y^2$ when $(x, y) \in C$ with the additional constraint $x + y = 2$.

5) Deduce the set of solutions of P.

Exercise 3

We consider the maximization problem

$$(P) \min(x^2 + xy - 2x + y^2 + z^2)$$

where x, y, z moves in $\mathbf{R}$.

Prove (precisely ! !) that there exists a unique solution, and compute it.

Exercise 4

A function from $\mathbf{R}$ to $\mathbf{R}$ is said to be quasiconvex if for every $(x, y) \in \mathbf{R}^2$ and every $\lambda \in [0, 1]$, $f(\lambda x + (1 - \lambda)y) \leq \max\{f(x), f(y)\}$, where $\max\{f(x), f(y)\}$ denotes the maximum between the two numbers $f(x)$ and $f(y)$.

a) Prove that any increasing function is quasiconvex.

(**We expect a precise proof.**)

b) Prove that $f(x) = -x^2$ is not quasiconvex.